České vysoké učení technické v Praze

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Interaction of a fluid flow with a deformable porous solid.

Summary. A continuum model is presented for fluid flow in a unsaturated deformable porous medium, where the fluid may undergo phase transitions with large changes of the specific volume. Typically, such problems arise in modeling water-ice-solid interactions in construction materials such as concrete, for example. The system of equations is derived from the conservation principles for mass, momentum, and energy, and from the Clausius-Duhem inequality for entropy. It couples the evolution of the displacement in the porous solid, of the capillary pressure, of the absolute temperature, and of the phase fraction. Mathematical results are proved under the additional hypothesis that inertia effects and shear stresses can be neglected. For the resulting nonlinear system of two PDEs, one ODE, and one ordinary differential inclusion with natural initial and boundary conditions, existence of global in time solutions is proved by means of cut-off techniques and a series of estimates independent of the cut-off parameters.

Interakce proudící kapaliny a deformovatelného porézního prostředí.

Souhrn. Je odvozen spojitý model pro proudící kapalinu v nenasyceném deformovatelném porézním prostředív situaci, kdy kapalina může vlivem fázového přechodu měnit skupenství z kapalného na pevné a naopak při velké změně specifického objemu. Podobné problémy typicky vznikají při matematickém modelování jevů souvisejících s prosakováním a zamrzáním vody ve stavebních materiálech, například v betonu. Soustava bilančních rovnic je odvozena ze zákonů zachování hmoty, hybnosti a energie a z Clausiovy-Duhemovy nerovnosti pro entropii a popisuje vzájemnou interakci veličin jako jsou deformace porézního materiálu, kapilární tlak, absolutní teplota a fázové proměnné. Matematické výsledky jsou dokázány za dodatečného předpokladu, že setrvačné jevy a smyková napětí mají zanedbatelný vliv na dynamiku procesu. Pro výslednou nelineární soustavu dvou parciálních diferenciálních rovnic, jedné obyčejné diferenciální rovnice a jedné diferenciální inkluze s přirozenými počátečními a okrajovými podmínkami je dokázána existence globálního řešení metodou ořezání rychle rostoucích nelinearit a sérií odhadů nezávislých na parametrech ořezání.

Klíčová slova: porézní prostředí, fázový přechod, parciální diferenciální rovnice, diferenciální inkluze.

Key words: porous media, phase transition, partial differential equations, differential inclusions.

Contents

Introduction

A model for fluid flow in partially saturated porous media with thermomechanical interaction was proposed and analyzed in [1, 3, 4]. We extend the model by including the effects of freezing and melting of the fluid in the pores. Typical examples, in which such situations arise, are related to groundwater flows and to the freezing-melting cycles of water sucked into the pores of concrete. Notice that the latter process forms one of the main reasons for the degradation of concrete in buildings, bridges, and roads. However, many of the governing effects in concrete like the multi-component microstructure, the breaking of pores, chemical reactions, and hysteresis of the saturation-pressure curves, and the occurrence of shear stresses, are still neglected in our model.

The modeling idea is the following. The pores in the matrix material contain a mixture of H_2O and gas, and H_2O itself is a mixture of the liquid (water) and the solid phase (ice). That is, in addition to the other physical quantities like capillary pressure, displacement, and absolute temperature, we need to consider the evolution of a phase parameter χ representing the relative proportion of water in the H_2O part and its influence on pressure changes due to the different mass densities of water and ice. Unlike in [1, 3, 4, 14], we do not consider hysteresis in the model. We believe that the mathematical results can be extended to the case of capillary hysteresis as in [1, 3, 4]. In our model without shear stresses, elastoplastic hysteresis effects as in [1, 3, 14] cannot occur.

As it will be detailed below, we assume that the deformations are small, so that $\operatorname{div} u$ is the relative local volume change, where u represents the displacement vector. Moreover, we assume that the volume of the matrix material does not change during the process, and thus the volume and mass balance equations with Darcy's law for the water flux lead to a nonlinear degenerate parabolic equation for the capillary pressure, see (2.4). In the equation of motion, we take into account the pressure components due to phase transition and temperature changes, and we further simplify the system in order to make it mathematically tractable by assuming that the process is quasistatic and the shear stresses are negligible. The problem of existence of solutions for the coupled system without this assumption is open and, probably, very challenging. Finally, we use the balance of internal energy and the entropy inequality to derive the dynamics for absolute temperature and phases; they turn out to be, respectively, a parabolic equation for the temperature with highly nonlinear right-hand side (quadratic in the derivatives) and an ordinary differential inclusion for the phase parameter χ.

Finally, let us note that – in order to model the freezing and melting phenomena in the pores – we use some ideas from publications on freezing and melting in containers filled with water with rigid, elastic, or elastoplastic boundaries (cf. [8, 9, 10, 11, 12]). It was shown there how important it is to account for the difference in specific volumes of water and of ice.

There is an abundant classical mathematical literature on phase transition processes, see, e. g., the monographs [2], [5], [15], and the references therein. It seems, however, that only few publications take into account the fact that the mass densities and specific volumes of the phases differ. In [6], the authors proposed to interpret a phase transition process in terms of a balance equation for macroscopic motions, and to include the possibility of voids. Well-posedness of an initial-boundary value problem associated with the resulting PDE system was proved there, and the case of two different densities ϱ_1 and ϱ_2 for the two substances undergoing phase transitions was pursued in [7].

1 The model

We consider a connected domain $\Omega \subset \mathbb{R}^3$ filled by a deformable matrix material with pores containing a mixture of H_2O and gas, where we assume that H_2O may appear in one of the two phases: water or ice. We also assume that the volume of the solid matrix remains constant during the process, and let $c_s \in (0,1)$ be the relative proportion of solid in the total reference volume. We denote, for $x \in \Omega$ and time $t \in [0,T]$,

$W(x,t) \in [0,1]$ ... relative proportion of H_2O in the total pore volume;

$A(x,t) \in [0,1]$ ... relative proportion of gas in the total pore volume;

$\chi(x,t) \in [0,1]$ ... relative proportion of water in the H_2O part;

$\xi(x,t)$ ... mass flux vector;

$p(x,t)$ ... capillary pressure;

$u(x,t)$ ... displacement vector;

$\sigma(x,t)$ ... stress tensor;

$\theta(x,t)$ ... absolute temperature.

Then χW represents the relative proportion of water in the total pore volume, and $(1-\chi)W$ represents the relative proportion of ice in the total pore volume.

We assume that the deformations are small, so that $\operatorname{div} u$ is the relative local volume change. By hypothesis, the volume of the matrix material does not change, so that the volume balance reads

$$W(x,t) + A(x,t) + c_s = 1 + \operatorname{div} u(x,t). \tag{1.1}$$

For A, we assume the functional relation

$$A = 1 - c_s - \varphi(p), \tag{1.2}$$

where φ is an increasing function that satisfies $\varphi(-\infty) = \varphi^\flat \in (0,1)$ and $\varphi(\infty) = 1 - c_s$, $\varphi^\flat + c_s < 1$. This means that the porous medium cannot be made completely dry by thermomechanical processes alone. Combining (1.1) with (1.2), we obtain that

$$W = \varphi(p) + \operatorname{div} u. \tag{1.3}$$

2 Mass balance

Consider an arbitrary control volume $V \subset \Omega$. The water content in V is given by the integral $\int_V \rho_L \chi W \,\mathrm{d}x$, where ρ_L is the water mass density, and the ice content is $\int_V \rho_S(1-\chi)W \,\mathrm{d}x$, where ρ_S is the ice mass density. The mass conservation principle then reads

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_L \chi W \,\mathrm{d}x + \int_{\partial V} \xi \cdot n \,\mathrm{d}s(x) = -\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_S(1-\chi)W \,\mathrm{d}x, \tag{2.1}$$

where n the unit outward normal vector to ∂V. In differential form, we obtain

$$\rho_L(\chi W)_t + \operatorname{div} \xi = -\rho_S((1-\chi)W)_t. \tag{2.2}$$

The right-hand side of (2.2) is the positive or negative liquid water source due to the solidification or melting of the ice. We assume the water flux in the form of the Darcy law

$$\xi = -\mu(p)\nabla p, \tag{2.3}$$

with a proportionality factor $\mu(p) > 0$. This, (1.3), and (2.2), yield the equation

$$\big((\chi{+}\rho^*(1{-}\chi))(\varphi(p) + \operatorname{div} u)\big)_t - \frac{1}{\rho_L}\operatorname{div}(\mu(p)\nabla p) = 0, \tag{2.4}$$

with $\rho^* = \rho_S/\rho_L \in (0,1)$.

3 Momentum balance

The equation of motion is considered in the form

$$\rho_M u_{tt} - \operatorname{div}\sigma = g, \tag{3.1}$$

where ρ_M is the mass density of the matrix material, σ is the stress tensor, and g is a volume force acting on the body (e. g., gravity). For σ, we prescribe the constitutive equation

$$\sigma = B\varepsilon_t + A\varepsilon + \big((\chi{+}\rho^*(1{-}\chi))(\lambda\operatorname{div} u - p) - \beta(\theta - \theta_c)\big)\delta, \tag{3.2}$$

where $\varepsilon = \nabla_s u := \frac{1}{2}(\nabla u + \nabla u^T)$ is the small strain tensor, δ is the Kronecker tensor, B is a symmetric positive definite viscosity tensor, A is the symmetric positive definite elasticity tensor of the matrix material, $\lambda > 0$ is the bulk elasticity modulus of water, $\theta > 0$ is the absolute temperature, $\theta_c > 0$ is a fixed referential temperature, and $\beta \in \mathbb{R}$ is the relative solid-liquid thermal expansion coefficient. The term $(\chi{+}\rho^*(1{-}\chi))(\lambda\operatorname{div} u - p)$ accounts for the pressure component due to the phase transition.

4 Energy and entropy balance

We have to derive formulas for the densities of internal energy U and entropy S such that the energy balance balance equation and the Clausius–Duhem inequality hold for all processes. Let q be the heat flux vector, and let $V \subset \Omega$ be again an arbitrary control volume. The total internal energy in V is $\int_V U\,\mathrm{d}x$, and the total mechanical power $Q(V)$ supplied to V equals

$$Q(V) = \int_V \sigma : \varepsilon_t\,\mathrm{d}x - \int_{\partial V} \frac{1}{\rho_L} p\,\xi\cdot n\,\mathrm{d}s(x),$$

where ξ is the fluid mass flux (2.3). We thus have that

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V U\,\mathrm{d}x+\int_{\partial V} q\cdot n\,\mathrm{d}s(x) = \int_V \sigma:\varepsilon_t\,\mathrm{d}x-\int_{\partial V}\frac{1}{\rho_L}p\,\xi\cdot n\,\mathrm{d}s(x). \quad (4.1)$$

Again, by the Gauss formula, we obtain the energy balance equation in differential form, namely

$$U_t+\operatorname{div} q = \sigma:\varepsilon_t-\frac{1}{\rho_L}\operatorname{div}(p\xi). \quad (4.2)$$

The internal energy and entropy densities U and S, as well as the heat flux vector q, have to be chosen in order to satisfy, for all processes, the Clausius–Duhem inequality

$$S_t+\operatorname{div}\left(\frac{q}{\theta}\right)\geq 0, \quad (4.3)$$

or, taking into account the energy balance (4.2),

$$U_t-\theta S_t+\frac{q\cdot\nabla\theta}{\theta}\leq\sigma:\varepsilon_t-\frac{1}{\rho_L}\operatorname{div}(p\xi). \quad (4.4)$$

We consider $\varepsilon,\chi,p,\theta$ as state variables and U,S as state functions, independent of $\nabla\theta$. Hence, as a consequence of (4.4), two inequalities have to hold separately for all processes, namely

$$q\cdot\nabla\theta\leq 0,\quad U_t-\theta S_t\leq\sigma:\varepsilon_t-\frac{1}{\rho_L}\operatorname{div}(p\xi). \quad (4.5)$$

For simplicity, we assume Fourier's law for the heat flux,

$$q=-\kappa(\theta)\nabla\theta, \quad (4.6)$$

with the heat conductivity coefficient $\kappa=\kappa(\theta)>0$. We further introduce the free energy F by the formula $F=U-\theta S$, so that, in terms of F, the second inequality in (4.5) takes the form

$$F_t+\theta_t S\leq\sigma:\varepsilon_t+\frac{1}{\rho_L}\operatorname{div}(p\mu(p)\nabla p). \quad (4.7)$$

We claim that the right choice of F for (4.7) to hold is given by

$$\begin{aligned} F &= \frac{1}{2}A\varepsilon:\varepsilon+(\chi+\rho^*(1-\chi))\left(V(p)+\frac{\lambda}{2}(\operatorname{div} u)^2\right)\\ &\quad +L\chi\left(1-\frac{\theta}{\theta_c}\right)-\beta(\theta-\theta_c)\operatorname{div} u+F_0(\theta)+I(\chi), \quad (4.8)\\ S &= -\frac{\partial F}{\partial\theta}=\frac{L}{\theta_c}\chi+\beta\operatorname{div} u-F_0'(\theta), \quad (4.9)\end{aligned}$$

where

$$V(p) = p\varphi(p) - \Phi(p), \quad \Phi(p) = \int_0^p \varphi(\tau)\,\mathrm{d}\tau, \tag{4.10}$$

$F_0(\theta)$ is a purely caloric component of F, $L > 0$ is the latent heat, and I is the indicator function of the interval $[0,1]$. It is easy to check that if we choose the phase dynamics equation in the form

$$\gamma(\theta)\chi_t + \partial I(\chi) \ni (1-\rho^*)\left(\Phi(p) + p\,\mathrm{div}\,u - \frac{\lambda}{2}(\,\mathrm{div}\,u)^2\right) + L\left(\frac{\theta}{\theta_c} - 1\right) \tag{4.11}$$

with a coefficient $\gamma(\theta) > 0$, then (4.7) holds for all processes. Now observe that

$$\begin{aligned} U &= F + \theta S \\ &= \frac{1}{2}A\varepsilon : \varepsilon + (\chi{+}\rho^*(1{-}\chi))\left(V(p) + \frac{\lambda}{2}(\,\mathrm{div}\,u)^2\right) \\ &\quad + L\chi + \beta\theta_c\,\mathrm{div}\,u + F_0(\theta) - \theta F_0'(\theta) + I(\chi). \end{aligned} \tag{4.12}$$

The derivative of the purely caloric component $F_0(\theta) - \theta F_0'(\theta)$ is the specific heat capacity $c(\theta) = -\theta F_0''(\theta)$. Assuming that $c(\theta) = c_0$ is a positive constant, we obtain that $F_0(\theta) = -c_0\theta\log(\theta/\theta_c)$ up to a linear function, and

$$\begin{aligned} U = \frac{1}{2}A\varepsilon : \varepsilon + (\chi{+}\rho^*(1{-}\chi))\left(V(p) + \frac{\lambda}{2}(\,\mathrm{div}\,u)^2\right) + L\chi \\ + \ \beta\theta_c\,\mathrm{div}\,u + c_0\theta + I(\chi). \end{aligned} \tag{4.13}$$

We now rewrite Eq. (4.2) in a more suitable form:

$$\begin{aligned} 0 &= U_t + \ \mathrm{div}\,q - \sigma : \varepsilon_t - \frac{1}{\rho_L}\,\mathrm{div}\,(p\mu(p)\nabla p) \\ &= (F + \theta S)_t + \ \mathrm{div}\,q - \sigma : \varepsilon_t - \frac{1}{\rho_L}\,\mathrm{div}\,(p\mu(p)\nabla p) \\ &= -B\varepsilon_t : \varepsilon_t - \frac{1}{\rho_L}\mu(p)|\nabla p|^2 - \gamma(\theta)\chi_t^2 + \theta S_t + \ \mathrm{div}\,q, \end{aligned} \tag{4.14}$$

which yields the identity

$$c_0\theta_t - \ \mathrm{div}\,(\kappa(\theta)\nabla\theta) = B\varepsilon_t{:}\varepsilon_t + \frac{1}{\rho_L}\mu(p)|\nabla p|^2 + \gamma(\theta)\chi_t^2 - \frac{L}{\theta_c}\theta\chi_t - \beta\theta\,\mathrm{div}\,u_t. \tag{4.15}$$

5 The mathematical problem

We consider the system

$$\big((\chi+\rho^*(1-\chi))(\varphi(p) + \operatorname{div} u)\big)_t = \frac{1}{\rho_L} \operatorname{div}(\mu(p)\nabla p), \tag{5.1}$$

$$\rho_M u_{tt} = \operatorname{div}\sigma + g, \tag{5.2}$$

$$\sigma = B\nabla_s u_t + A\nabla_s u + ((\chi+\rho^*(1-\chi))(\lambda \operatorname{div} u - p) - \beta(\theta-\theta_c))\delta, \tag{5.3}$$

$$\gamma(\theta)\chi_t + \partial I(\chi) \ni (1-\rho^*)\Big(\Phi(p) + p \operatorname{div} u - \frac{\lambda}{2}(\operatorname{div} u)^2\Big) + \frac{L}{\theta_c}(\theta-\theta_c), \tag{5.4}$$

$$c_0\theta_t - \operatorname{div}(\kappa(\theta)\nabla\theta) = \frac{1}{\rho_L}\mu(p)|\nabla p|^2 + \gamma(\theta)\chi_t^2 - \frac{L}{\theta_c}\theta\chi_t + B\nabla_s u_t : \nabla_s u_t - \beta\theta \operatorname{div} u_t, \tag{5.5}$$

for the unknown functions p, u, χ, θ, coupled with the boundary conditions

$$u = 0, \tag{5.6}$$

$$\xi \cdot n = \alpha(x)(p - p^*), \tag{5.7}$$

$$q \cdot n = \omega(x)(\theta - \theta^*), \tag{5.8}$$

on $\partial\Omega$, where p^* is a given outer pressure, θ^* is a given outer temperature, $\alpha(x) \geq 0$ is the permeability of the boundary, and $\omega(x) \geq 0$ is the heat conductivity of the boundary.

We now simplify the problem by assuming that water is incompressible, that is, $\lambda = 0$. A further simplification consists in assuming that the process is quasistatic and that the shear stresses are negligible. Then (5.2)–(5.3) can be reduced to

$$0 = \operatorname{div}\sigma + g, \tag{5.9}$$

$$\sigma = (\nu \operatorname{div} u_t + \lambda_M \operatorname{div} u - p(\chi+\rho^*(1-\chi)) - \beta(\theta - \theta_c))\delta. \tag{5.10}$$

Assuming that the force g admits a potential G, that is, $g = \nabla G$, this yields

$$\nu \operatorname{div} u_t + \lambda_M \operatorname{div} u - p(\chi+\rho^*(1-\chi)) - \beta(\theta - \theta_c) = -G + H(t), \tag{5.11}$$

where $H(t)$ is an "integration constant", ν is the bulk viscosity coefficient, and λ_M is the bulk elasticity modulus of the matrix material.

In view of the boundary condition (5.6), we have that

$$H(t) \;=\; -\frac{1}{|\Omega|}\int_\Omega (p(\chi{+}\rho^*(1{-}\chi)) + \beta(\theta-\theta_c) - G)(x,t)\,\mathrm{d}x. \qquad (5.12)$$

With the new unknown function $w = \operatorname{div} u$, which represents the *relative volume change*, the system (5.1)–(5.5) then becomes

$$\big((\chi{+}\rho^*(1{-}\chi))(\varphi(p){+}w)\big)_t = \frac{1}{\rho_L}\operatorname{div}(\mu(p)\nabla p), \qquad (5.13)$$

$$\nu w_t + \lambda_M w = p(\chi{+}\rho^*(1{-}\chi)) + \beta(\theta-\theta_c) - G + H(t), \qquad (5.14)$$

$$\gamma(\theta)\chi_t + \partial I(\chi) \ni (1{-}\rho^*)\,(\Phi(p) + pw) + L\left(\frac{\theta}{\theta_c} - 1\right), \qquad (5.15)$$

$$c_0\theta_t - \operatorname{div}(\kappa(\theta)\nabla\theta) = \nu w_t^2 + \frac{1}{\rho_L}\mu(p)|\nabla p|^2 + \gamma(\theta)\chi_t^2 - \frac{L}{\theta_c}\theta\chi_t - \beta\theta w_t. \qquad (5.16)$$

We prescribe the initial conditions

$$p(x,0) \;=\; p^0(x), \qquad (5.17)$$
$$w(x,0) \;=\; w^0(x), \qquad (5.18)$$
$$\chi(x,0) \;=\; \chi^0(x), \qquad (5.19)$$
$$\theta(x,0) \;=\; \theta^0(x). \qquad (5.20)$$

6 Main results

We make the following hypothesis on the data.

Hypothesis 6.1 We fix a time interval $[0,T]$ and assume that the data of Problem (5.13)–(5.20) have the following properties:

(i) $\gamma : [0,\infty) \to [0,\infty)$ is continuous; $\exists 0 < c_\gamma < C_\gamma : c_\gamma(1+\theta) \le \gamma(\theta) \le C_\gamma(1+\theta)$ for all $\theta \ge 0$;

(ii) $\kappa : [0,\infty) \to [0,\infty)$ is continuous; $\exists 0 < c_\kappa < C_\kappa$, $0 < a < 1$, $a < \hat{a} < \frac{16}{5} + \frac{6}{5}a : c_\kappa(1+\theta^{1+a}) \le \kappa(\theta) \le C_\kappa(1+\theta^{1+\hat{a}})$ for all $\theta \ge 0$;

(iii) $\theta^0 \in W^{1,2}(\Omega) \cap L^\infty(\Omega)$, $\theta^* \in L^\infty(\partial\Omega \times (0,T))$, $\theta^*_t \in L^2(\partial\Omega \times (0,T))$, $\exists \bar{\theta} > 0 : \theta^0(x) \ge \bar{\theta}$, $\theta^*(x,t) \ge \bar{\theta}$;

(iv) $\exists\, 0 < \hat{\delta} \le \delta < 1/4$, $\exists\, 0 < c_\varphi < C_\varphi$ such that for all $p \in \mathbb{R}$ we have that
$c_\varphi \max\{1,|p|\}^{-1-\delta} \le \varphi'(p) \le C_\varphi \max\{1,|p|\}^{-1-\hat{\delta}}$;

(v) $\exists 0 < c_\mu < C_\mu : c_\mu \le \mu(p) \le C_\mu$ for all $p \in \mathbb{R}$;

(vi) $p^0 \in W^{1,2}(\Omega) \cap L^\infty(\Omega)$, $p^* \in L^\infty(\partial\Omega \times (0,T)) \cap L^2(0,T;W^{1,2}(\partial\Omega))$, $p^*_t \in L^2(\partial\Omega \times (0,T))$;

(vii) $w^0, \chi^0 \in L^\infty(\Omega)$, $\chi^0(x) \in [0,1]$ a. e., $\int_\Omega w^0(x)\,\mathrm{d}x = 0$;

(viii) $G \in L^\infty(\Omega \times (0,T))$, $G_t \in L^2(\Omega \times (0,T))$;

(ix) $\Omega \subset \mathbb{R}^3$ is a bounded connected set of class $C^{1,1}$, $\alpha : \partial\Omega \to [0,\infty)$ is Lipschitz continuous, $\omega \in L^\infty(\partial\Omega)$, $\omega(x) \ge 0$ a. e., $\int_{\partial\Omega} \alpha(x)\,\mathrm{d}s(x) > 0$, $\int_{\partial\Omega} \omega(x)\,\mathrm{d}s(x) > 0$.

The main result is the following existence theorem, and its proof can be found in [13].

Theorem 6.2 *Let Hypothesis 6.1 hold true. Then there exists a solution* (p,w,χ,θ) *to the system* (5.12)–(5.20) *with the regularity*

$$p \in L^\infty(\Omega\times(0,T)),\ p_t, \nabla\theta \in L^2(\Omega\times(0,T)),\ \nabla p \in L^\infty(0,T;L^2(\Omega)), \tag{6.1}$$

$$\theta, w_t \in L^{\bar{p}}(\Omega \times (0,T)),\ w, \chi_t \in L^\infty(0,T;L^{\bar{p}}(\Omega)) \text{ for } \bar{p} < 8+a, \tag{6.2}$$

$$\theta_t \in L^2(0,T;W^{-1,q^*}(\Omega)) \text{ with some } q^* > 1. \tag{6.3}$$

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03 - 05/2018: visiting professor, University of Modena, Italy

10 - 11/2016: visiting professor, Hohai University, Nanjing, China

02 - 04/2016: visiting professor, INSA Lyon, France

06 - 07/2014: Compact PhD course, TU Munich, Germany

09 - 10/2013: visiting professor, INSA Lyon, France

04 - 07/2010: John-von-Neumann-Gastprofessur, TU Munich, Germany

05/2003: PhD and postdoc course, Università di Pavia, Italy

03 - 06/1995: visiting professor, Université de Technologie de Compiègne, France

09 - 12/1990: visiting professor, University of Wisconsin at Milwaukee, U.S.A.

Organization of large congresses:

Head of the organizing committee EQUADIFF 13, Prague, 2013, 370 participants

Publications: 2 monographs, 120 papers in refereed journals, 34 conference papers, 1119 WoS citations without self-citations, HI 17, 1 Patent No. 10 2007 001 186 of the German Patent and Trade Mark Office

Awards:

Research Award of the Minister of Education of the Czech Republic, 2001

Bernard Bolzano Honorary Medal for Merit in Mathematical Sciences, 2014

List of publications 2012–2019

[1] P. Krejčí, M. Al Janaideh, F. Deasy: Inversion of hysteresis and creep operators. *Physica B: Condensed Matter* **407** (2012), 1354–1356.

[2] M. Eleuteri, J. Kopfová, P. Krejčí: A thermodynamic model for material fatigue under cyclic loading. *Physica B: Condensed Matter* **407** (2012), 1415–1416.

[3] M. Al Janaideh, P. Krejčí: Prandtl-Ishlinskii hysteresis models for complex time dependent hysteresis nonlinearities. *Physica B: Condensed Matter* **407** (2012), 1365–1367.

[4] P. Krejčí, J.P. O'Kane, A. Pokrovskii, D. Rachinskii: Properties of solutions to a class of differential models incorporating Preisach hysteresis operator. *Physica D: Nonlinear Phenomena* **241** (2012), 2010–2028.

[5] P. Krejčí: The Preisach hysteresis model: Error bounds for numerical identification and inversion. *Discrete Cont. Dyn. Systems S* **6** (2013), 101–119.

[6] P. Krejčí, E. Rocca: Well-posedness of an extended model for water-ice phase transitions. *Discrete Cont. Dyn. Systems S* **6** (2013), 439–460.

[7] M. Brokate, P. Krejčí: Optimal control of ODE systems involving a rate independent variational inequality. *Discrete Cont. Dyn. Systems B* **18** (2013), 331–348.

[8] M. Eleuteri, J. Kopfová, P. Krejčí: Fatigue accumulation in an oscillating plate. *Discrete Cont. Dyn. Systems S* **6** (2013), 909–923.

[9] M. Eleuteri, J. Kopfová, P. Krejčí: Non-isothermal cyclic fatigue in an oscillating elastoplastic beam. *Communications on Pure and Applied Analysis* **12** (2013), 2973–2996.

[10] D. Davino, P. Krejčí, C. Visone: Fully coupled modeling of magnetomechanical hysteresis through 'thermodynamic' compatibility. *Smart Materials and Structures* **22** (2013), 095009.

[11] P. Krejčí, A. Petrov: Existence and uniqueness results for a class of dynamic elasto-plastic contact problems. *J. Math. Anal. Appl.* **408** (2013), 125–139.

[12] S. Frigeri, M. Grasselli, P. Krejčí: Strong solutions for two-dimensional nonlocal Cahn – Hilliard – Navier – Stokes systems. *J. Diff. Eq.* **255** (2013), 2587–2614.

[13] M. Al Janaideh, P. Krejčí: Inverse rate-dependent Prandtl-Ishlinskii model for feedforward compensation of hysteresis in a piezomicropositioning actuator. *IEEE-ASME Transactions on Mechatronics* **18** (2013), 1498–1507.

[14] A. H. El-Shaer, M. Al Janaideh, P. Krejčí, M. Tomizuka: Robust performance enhancement using disturbance observers for hysteresis compensation based on generalized Prandtl-Ishlinskii model. *Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME* **135** (5), 2013, 051008.

[15] M. Al Janaideh, R. Naldi, R. Marconi, P. Krejčí: A hybrid model for the play hysteresis operator. *Physica B: Condensed Matter* **430** (2013), 95–98.

[16] S. Bosia, M. Eleuteri, J. Kopfová, P. Krejčí: Fatigue and phase transition in an oscillating plate. *Physica B* **435** (2014), 1–3.

[17] P. Colli, G. Gilardi, P. Krejčí, J. Sprekels: A vanishing diffusion limit in a nonstandard system of phase field equations. *Evolution Equations and Control Theory* **3** (2014), 257–275.

[18] P. Krejčí, V. Recupero: Comparing BV solutions of rate independent processes. *Journal of Convex Analysis* **21** (2014), 121–146.

[19] P. Colli, G. Gilardi, P. Krejčí, J. Sprekels: A continuous dependence result for a nonstandard system of phase field equations. *Mathematical Methods in the Applied Sciences* **37** (2014), 1318–1324.

[20] P. Colli, G. Gilardi, P. Krejčí, P. Podio-Guidugli, J. Sprekels: Analysis of a time discretization scheme for a nonstandard viscous Cahn-Hilliard system. *ESAIM: Mathematical Modelling and Numerical Analysis* **48** (2014), 1061–1087.

[21] P. Krejčí, H. Lamba, S. Melnik, D. Rachinskii: Analytical solution for a class of network dynamics with mechanical and financial applications. *Physical Review E* **90** (2014), 032822-.

[22] M. Eleuteri, J. Kopfová, P. Krejčí: Fatigue accumulation in a thermo-visco-elastoplastic plate. *Discrete Cont. Dyn. Systems B* **19** (2014), 2091–2109.

[23] M. Brokate, P. Krejčí: Weak differentiability of scalar hysteresis operators. *Discrete Cont. Dyn. Systems A* **35** (2015), 2405–2421.

[24] M. Eleuteri, J. Kopfová, P. Krejčí: A new phase field model for material fatigue in an oscillating elastoplastic beam. *Discrete Cont. Dyn. Systems A* **35** (2015), 2465–2495.

[25] P. Krejčí, A. Petrov: Elasto-plastic contact problems with heat exchange. *Nonlinear Analysis: Real World Applications* **22** (2015), 551–567.

[26] P. Krejčí, A. A. Tolstonogov, S. A. Timoshin: A control problem in phase transition modeling. *Nonlinear Differential Equations and Applications NoDEA* **22** (2015), 513–542.

[27] P. Krejčí, H. Lamba, S. Melnik, D. Rachinskii: Kurzweil integral representation of interacting Prandtl-Ishlinskii operators. *Discrete Cont. Dyn. Systems B* **20** (2015), 2949–2965.

[28] B. Albers, P. Krejčí: Unsaturated porous media flow with thermomechanical interaction. *Mathematical Methods in the Applied Sciences* **39** (2016), 2220–2238.

[29] P. Krejčí, H. Lamba, G. A. Monteiro, D. Rachinskii: The Kurzweil integral in financial market modeling. *Mathematica Bohemica* **141** (2016), 261–286.

[30] D. Davino, P. Krejčí, A. Pimenov, D. Rachinskii, C. Visone: Analysis of an operator-differential model for magnetostrictive energy harvesting. *Communications in Nonlinear Science and Numerical Simulation* **39** (2016), 504–519.

[31] M. Al Janaideh, D. Davino, P. Krejčí, C. Visone: Comparison of Prandtl-Ishlinskiĭ and Preisach modeling for smart devices applications. *Physica B: Condensed Matter* **486** (2016), 155–159.

[32] B. Kaltenbacher, P. Krejčí: A thermodynamically consistent phenomenological model for ferroelectric and ferroelastic hysteresis. *ZAMM – Z. Angew. Math. Mech.* **96** (2016), 874–891.

[33] P. Krejčí, S. A. Timoshin: Coupled ODEs control system with unbounded hysteresis region. *SIAM J. Control Optim.* **54** (2016), 1934–1949.

[34] B. Detmann, P. Krejčí, E. Rocca: Solvability of an unsaturated porous media flow problem with thermomechanical interaction. *SIAM J. Math. Anal.* **48** (2016), 4175–4201.

[35] W. Liu, P. Krejčí, G. Ye: Continuity properties of Prandtl-Ishlinskii operators in the space of regulated functions. *Discrete and Continuous Dynamical Systems Series B* **22** (2017), 3783–3795.

[36] P. Krejčí, E. Rocca, J. Sprekels: Unsaturated deformable porous media flow with thermal phase transition. *Mathematical Models and Methods in Applied Sciences* **27** (2017) 2675–2710.

[37] P. Krejčí, S. A. Timoshin, A. A. Tolstonogov: Relaxation and optimisation of a phase-field control system with hysteresis. *International Journal of Control* **91** (2018), 85–100.

[38] P. Krejčí, A. Petrov: A mathematical model for the third-body concept. *Mathematics and Mechanics of Solids* **23** (3) (2018), 420–432.

[39] F. Dabaghi, P. Krejčí, A. Petrov, J. Pousin, Y. Renard: A weighted finite element mass redistribution method for dynamic contact problems. *Journal of Computational and Applied Mathematics* **345** (2019), 338–356.

[40] P. Krejčí, G. A. Monteiro: Oscillations of a temperature-dependent piezoelectric rod. *Nonlinear Analysis: Real World Applications* **46** (2019), 403–420.