Report on Morphisms, Infinite words and symmetries by Štěpán Starosta

February 24, 2017

The report presented by Štěpán Starosta for his Habilitation Thesis is formed of a 30 pages introduction and ten papers already published or submitted for publication (in general coauthored with others).

The introduction is a very well written summary of the contents of the following papers, put in perspective of their role in the domain of discrete mathematics and combinatorics on words. As explained in the abstract preceding it, many of the papers are related to conjectures in the domain that they either completely solve (the Brlek-Reutenauer conjecture) or partially solve (the Class P Conjecture and the Zero Defect Conjecture). This represents for me a strong point of the work presented.

In the first paper (Ľ. Balková, E. Pelantová and Š. Starosta, Proof of Brlek-Reutenauer conjecture , Theoret. Comput. Sci. 475 (2013), 120125), the authors give a proof of the Brlek-Reutenauer conjecture relating the defect $D(u)$ of an infinite word u, its factor complexity $C(u)$ and its palindromic complexity $P(u)$ by the formula $2D(u) = \sum \Delta(C(u) + P(u))$. The defect of an infinite word is the maximum of the defect of its prefixes and the defect of a word is the difference $|w| + 1 - P(w)$ wher $P(w)$ is the number of palindrome factors of w. The conjecture had been first proved by Brlek and Reutenauer for periodic words (2011), and then by the same group of authors for uniformly recurrent words (2012). The proof had a flaw found by Bojan Bašic (TCS, 2012) and corrected by the authors (TCS, 2012).

In the second paper (Z. Masáková, E. Pelantová and Š. Starosta, Itineraries induced by exchange of three intervals, Acta Polytechnica, Vol. 56, No. 6 (2016)), the authors study exchanges of three intervals. They prove that the words coding a non-degenerate three interval exchange satisfy the Class $\mathbb{P}$ conjecture. This conjecture, proposed by Hof, Knill and Simon in 1995 asserts that for a fixed point u of a primitive morphism containing infinitely many palindromic factors, there exists a morphism of class $\mathbb{P}'$ such that its fixed point has the same factors as u. A morphism $\varphi : A^* \to A^*$ belongs to class $\mathbb{P}$ if there is a palindrome word p such that for every letter $a \in A$, one has $\varphi(a) = pp_a$ where p_a is a palindrome. It is in class $\mathbb{P}'$ if it is conjugate to a morphism in $\mathbb{P}$.

Université de Marne-la-Vallée, F-77454 Marne-la-Vallée Cedex 2 - Tél. : +33 1 60 95 75 65 - Fax : +33 1 60 95 75 57
www-igm.univ-mlv.fr

In the third paper (S. Labbé, E. Pelantová and Š. Starosta, On the Zero Defect Conjecture, Eur. J. Combin. 62 (2017), 132-146), the authors prove the validity in several cases of the Zero Defect conjecture. This conjecture, proposed in 2008 by Blondin Massé, Brlek, Garon and Labbé asserts that if u is a fixed point of a primitive morphism such that $D(u) < \infty$, then u is periodic or $D(u) = 0$. A counterexample on a ternary alphabet was found by Bucci and Vaslet in 2012. The authors prove that the conjecture is true on a binary alphabet and also for a particular class of morphisms called marked morphisms.

The fourth paper (Morphic images of episturmian words having finite palindromic defect, Eur. J. Combin. 51 (2016), 359371) is authored by Š. Starosta alone. It is rather technical. The two main results of the paper give a family of morphisms sending any episturmian word on a rich or almost rich word. An finite word is rich if it has zero defect. An infinite word is rich if every factor is rich and almost rich if $D(u)$ is finite. The classes of morphisms studied are

- a class P_{ret} of morphisms introduced in another paper (L. Balková, E. Pelantová, Š. Starosta, Infinite words with finite defect, Adv. Appl. Math. 47 (3) (2011) 562574) and related to return words.
- The morphisms called binary projections which send letters on their class in a partition of the alphabet in two subsets.

The next paper presented (Pelantová, Edita and Starosta, Štepán - Constructions of words rich in palindromes and pseudopalindromes, Discrete Mathematics & Theoretical Computer Science, November 22, 2016, Vol. 18 no. 3). The pseudopalindromes have been introduced in another paper (E. Pelantová and Š. Starosta. Languages invariant under more symmetries: overlapping factors versus palindromic richness. Discrete Math., 313:24322445, 2013) not included in the selection presented for the Habilitation. It generalizes classical palindromes by allowing symmetries on the alphabet and, more generally, the action of a group G of morphisms and antimorphisms (like the mirror image). This leads to the definition of G-rich words where palindromes are replaced by the more general notion of pseudopalindromes. The article presents constructions defining G-rich words and relating Sturmain words (the classical example of rich words) with another class introduced by Rote (1993).

The sixth paper (T. Jajcayová, E. Pelantová, Š. Starosta, Palindromic closures using multiple antimorphisms, Theoret. Comput. Sci. 533 (2014), 3745) deals with generalized pseudostandard words, a notion introduced by Aldo De Luca and Alessandro De Luca in 2006. The main result is that generalized Thue-Morse words $t_{b,m}$ are generalized pseudostandard words for $b \leq m$. These words are defined by $t_{b,m} = (s_b(n) \bmod m)_{n\geq 0}$ where $s_b(n)$ is the sum of the digits of the representation of n in base b.

The seventh and eight papers (An Algorithm Enumerating All Infinite Repetitions in a D0L-System, Journal of Discrete Algorithms 33 (2015), 130138) and (Characterization of circular D0L-systems, submitted) coauthored with K. Klouda, are a substantial contribution to the understanding of the notion of recognizability of morphisms. This notion was introduced by an early paper of

J.C Martin in 1973 and led to an important theorem due to B. Mossé in 1992. Later, Mignosi and Seebold showed a link between recognizability and repetitions in DOL systems. The contribution of Klouda and Starosta represents a further step in the understanding of these difficult notions.

The last two papers are of a different kind and serve, to use Stepan Starosta's terms "as an illustration of connection of Combinatorics on Words to other research domains". The first one (P. Arnoux and Š. Starosta, The Rauzy gasket, in: J. Barral and S. Seuret (Eds.), Further Developments in Fractals and Related Fields, Trends in Mathematics 2013, pp. 1 - 23, Springer Science+Business Media New York 2013) concerns the set , called Rauzy gasket, of frequencies of episturmian ternary words. This defines a fractal subset of $\mathbb{R}^3$ which is compared with other classical fractal sets such as the Sierpiński gasket. The second one (E. Pelantová, Š. Starosta, M. Znojil, Markov constant and quantum instabilities, J. Phys. A: Math. Theor. 49 155201) relates combinatorics on words with number theory (Markov constants).

The set of papers presented in rich and interestingly focused on a bunch of difficult problems in combinatorics on words. They represent an important contribution to the understanding of these problems. I am therefore quite convinced that this is more than the usual contents required for a Habilitation.

Dominique Perrin