MUNI
SCI

Habilitační komise
Fakulta jaderná a fyzikálně inženýrská
České vysoké účení technické v Praze

Report on

The habilitation thesis
ALGEBRAIC PROBLEMS IN QUANTUM MECHANICS ON GRAPHS
by **Ing. Ondřej Turek, PhD.**

I consider it an honour to be asked to write a report on a habilitation thesis that is so well written and interesting as the work of Ondřej Turek. Out of a long publication list the author has selected ten papers published in peer reviewed international journals with impact factors and the thesis is a commentary on these publications.

The thesis is concerned with mathematical and physical properties of so called Quantum Graphs. A Quantum Graph is a linear, network-shaped structure of vertices connected by edges in which each edge is given a length and where a Schrödinger equation is imposed on each edge. These kind of structures have been used in physics as models of free electrons in organic molecules or in the study of waveguides, in photonic crystals and in Anderson localization.

After an introductory chapter where Quantum Graphs and some related mathematical structures are defined the thesis introduces us to three different applications in the field. In the first two parts the local properties of a Quantum Graph are studied. One vertex is singled out and all possible structures that can appear are classified and discussed. It is shown how to construct Quantum Gates that can be controlled to filter the frequencies of a signal in various ways. The chapter is comprehensively written and the reader is not left with any unclear points or a feeling that there is more to be said.

In the next section the same problems as in the previous section are dealt with but from a mathematical point of view. In particular the mathematical properties of the matrices describing the vertices of the graph are connected to the physical properties of the graph. It is interesting to see how mathematics and physics goes hand in hand and how physical thinking can lead to progress in mathematics and vice versa.

In the last section we are dealing with the global properties of periodic Quantum Graphs in that the question about the nature of the spectrum of such graphs is discussed. Here the discussion becomes a little bit more sketchy and it is not possible to follow all the details of the proofs. This is presumably since the mathematics needed is considerably more involved when dealing with the spectrum of the whole infinite graph than with the scattering properties of a single vertex but it would have been nice to read a thorough introduction to the Floquet-Bloch decomposition since the previous chapters has shown that the author has a talent for pedagogical writing.

The results described in the text are backed up by ten appendices consisting of the papers where the

Masaryk University, Faculty of Science, Department of Theoretical Physics and Astrophysics

Kotlářská 267/2, 611 37 Brno, Česká republika
T: +420 549 49 8084, E: unge@physics.muni.cz, W: www.sci.muni.cz

original results are published.

As said before, the text is nicely written and enjoyable to read. The level of the english is excellent. However, sometimes it shows that pieces of text have been copied from papers where a different notation is used. For instance, on page 45 the author talks about the $m = 3$ case although I believe he means $r = 3$ (m is the notation used in the original paper). I however do not consider this to be in any way taking merit away from this excellent thesis. If there is time during the defense I would instead like to ask the following questions:

- How does the spectrum of periodic quantum graphs depend on the dimension? Are two dimensional graphs special in any way?
- Has anyone tried to model high T_c superconductors using quantum graphs? Here there are two dimensional structures that are conducting but only weakly interacting between themselves.

The habilitation thesis entitled "Algebraic Problems in Quantum Mechanics on Graphs" by Ing. Ondřej Turek, PhD. by far fulfils the requirements expected of a habilitation thesis in Mathematical Physics. Ing. Turek is a well established and respected researcher in the field of Mathematical Physics and I fully and enthusiastically support his application to become Associate Professor (to be given the title docent).

Rikard von Unge
Professor of Theoretical Physics and
Astrophysics