

March 29, 2018

Evaluation report on the habilitation thesis of Dr. Milan Kucharik

Numerical Methods for Quantity Remapping in the Context of Indirect Arbitrary Lagrangian-Eulerian (ALE) Hydrodynamics

This cumulative habilitation thesis summarizes Dr. Milan Kucharik's recent work on the development of physics-compatible conservative interpolation (remapping) techniques for Arbitrary Lagrangian-Eulerian (ALE) methods. In contrast to monolithic ALE formulations which are well suited for simulating small deformations of smooth fluid-solid interfaces, the methodology employed in this thesis belongs to the family of indirect ALE schemes designed to handle arbitrary deformations and interactions of multiple materials (fluids or solids) in a robust and realistic manner. The thesis begins with a concise introduction to the indirect ALE methodology which involves a Lagrangian mesh update, a rezoning step, and remapping of data onto the new mesh. The methods proposed in the ten journal articles that form the core of this thesis provide new tools for overcoming numerical difficulties that arise when it comes to the design of high-order schemes for constrained remapping (especially in applications to systems of conserved quantities, evolving interfaces, and interacting materials).

The main accomplishments of this thesis, as summarized in Section 4.11, include groundbreaking contributions to the design and analysis of new remapping algorithms utilizing swept-based and intersection-based approaches to calculation of numerical fluxes. The improvements proposed by Dr. Kucharik make it possible to use meshes with changing connectivity, combine different kinds of remapping so as to exploit their complementary advantages, and ensure preservation of essential physical properties (symmetry, conservation of kinetic energy) in remapping schemes for velocity fields. Moreover, the hybrid remapping concept introduced by Dr. Kucharik and his collaborators leads to dramatic gains in efficiency since expensive calculation of intersection-based fluxes is restricted to a neighborhood of material interfaces, whereas the computationally efficient swept-region version is employed elsewhere.

The practical value of this impressive work goes far beyond the use of proposed methods in indirect ALE schemes and applications to laser plasma hydrodynamics which are reviewed in Chapter 5. As a matter of fact, many ideas appear to have significant merit in the context of unstructured grid finite element methods for compressible gas dynamics. For this reason, I have been following the work of Dr. Kucharik and his coworkers with great interest in recent years. In particular, I believe that flux-corrected remapping (FCR) schemes presented in Papers 7 and 8 will also significantly advance the state of the art in the design of flux-corrected transport (FCT) algorithms and other invariant domain preserving limiting techniques for finite element discretizations of the compressible Euler and ideal MHD equations. While reading Paper 7 in my capacity as reviewer of this thesis, I suddenly realized that the FCR algorithm presented in Section 4.5 (see also Section 5 of Paper 8) defines the momentum fluxes for bounds-preserving remap in essentially the same way as my coworkers and I do in a more recent paper on sequential FCT limiting for the Euler equations. A key advantage of this definition is the possibility of limiting the mass (density) in a segregated manner without producing spurious undershoots/overshoots in other quantities of interest (velocity, specific total energy, volume fractions etc.) On top of that, Dr. Kucharik's FCR algorithms are symmetry-preserving and handle the kinetic energy in a way which guarantees conservation of total energy. At the moment, I am not aware of any high-resolution Eulerian or monolithic ALE scheme that would provide all of these properties on general meshes. However, I envisage that such schemes could be developed using physics-aware limiting tools based on Dr. Kucharik's ideas.

Discretization, remapping, and limiting techniques that preserve so many qualitative properties of the exact solution simultaneously without generating inordinate amounts of artificial diffusion are extremely rare and difficult to develop. Dr. Kucharik's contributions to the field are highly visible and inspiring. The introductory part of the thesis is very well written and provides an excellent roadmap to the original publications. The author's own contributions to joint work are clearly identified in Section 4. There is no doubt that Dr. Kucharik played a leading role in conducting the presented research. Papers 9 and 10 are coauthored by his PhD student Matej Klima. The high quality of these publications (as well as several talks by Mr. Klima including the one I invited him to give at our university in March 2017) demonstrate Dr. Kucharik's ability to supervise PhD students leading them all the way to the frontiers of current research and teaching them to solve difficult real-life problems. In light of the above, it is my pleasure to recommend that this outstanding habilitation thesis be **accepted**.

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