Referee's report on the Habilitation thesis

MORPHISMS, INFINITE WORDS, AND SYMMETRIES

by Dr. Štěpán Starosta

The thesis comprises 10 research papers by the author (nine of them are joint work with one or two coauthors each; the list of coauthors consists of L. Balková, E. Pelantová, Z. Masáková, S. Labbé, T. Jajcayová, K. Klouda, P. Arnoux, and M. Znojil). These papers constitute the Chapters 2–11 of the thesis. The first chapter is a nicely written overview of the papers which places them in a proper perspective inside the area of combinatorics on words.

Seven out of the ten papers are published and three are submitted. Six papers are published in authoritative international journals (five in the area of discrete mathematics and theoretical computer science, and one in the area of theoretical physics) and one in the Springer proceedings of an international conference. All the papers in the thesis meet the standards of a quality scientific publication.

Eight out of ten papers solve problems from combinatorics on words, one links combinatorics on words to fractals, and the last one establishes some number-theoretic facts motivated by certain problems in physics; nevertheless, even in this paper some combinatorics on words is used in the proofs. So the thesis in general is unified by the study of combinatorial properties of words (mostly right-infinite ones). Some ideas, constructions, methods are used in several papers.

The biggest pool of results in the thesis (papers [I, III-VI]) is related to palindromic richness of right-infinite words; the paper [II] is also partially related to palindromes but in a different context. I am quite satisfied with the picture obtained in these papers for the properties of richness and almost richness of infinite words both for ordinary palindromes and for involutive palindromes.

My favourite paper in this thesis is [IX], mainly not because of the beautiful pictures but because of a nice connection between a seemingly unrelated areas: combinatorics on words and fractal theory. The authors[1] introduce and study a fractal subset of a two-dimensional simplex, formed by triples of possible densities of letters in the ternary Arnoux–Rauzy words and called

[1] This is a joint paper.

the Rauzy gasket. The paper clearly demonstrates deep connections between different branches of mathematics.

All papers are well-written, as well as the introduction. All proofs are given in sufficient detail for verification. I point out a big number of well-chosen examples accompanying non-trivial notions and illustrating the results.

At the same time, I see some points for mild criticism.

- Several papers mention results of computer experiments: but there are no references to program code or, at least, discussion of algorithms and there performance. In paper [VII], an algorithm is the main result, but again, nothing is said about its theoretical and practical efficiency.

- Speaking about marked morphisms [III] and circularity criteria [VIII], it is necessary to mention the results of Anna Frid from the end of 90s on factor complexity, recurrence function and other parameters of the D0L words generated by marked morphisms. I mention here just one her result: a fixed point of a uniform marked morphism is non-circular if and only if it is similar (in some natural sense) to the Sierpinsky word.

- While the list of "hot" topics in combinatorics on words, given in the introduction, is big enough, some topics are clearly missing. One example is the repetitions in finite words (runs conjecture and beyond; maximum number of squares; expected number of runs, squares, palindromes, gapped repeats, gapped palindromes, ...). Another example is combinatorics of partial words, studying numerous similarities and distinctions between partial and "usual" words. Also, in the links to other domains, stringology, with its lots of connections to and from combinatorics on words, is not even mentioned.

- Finally, there are some misprints in formulas in the introduction: for example, '+' instead of '−' in the Brlek–Reutenauer formula in pp. 12–13 can mislead the reader.

The above criticism affects only some points in the presentation but not the results. The general impression of Dr. Starosta's work is highly positive. Summarizing I can say that:

- the thesis contains several novel scientific results of high level presented in a good style;

- the thesis clearly demonstrates that Dr. Starosta as both a talented scientist and a mature scientific writer;

- altogether, the thesis proves that Dr. Starosta masters the science of mathematics and is able to further promote it.

I recommend the acceptance of the thesis.

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