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Referee report on the Habilitation thesis
"Multivariate Fourier-Weyl Transformation and their Application"
by Jiří Hrivnák

The symmetries described by classical Lie groups and Lie algebras are present in various physical systems. Relations among special functions related to those symmetries can be derived from Weyl orbit functions that that represent a multivariable generalization of classical trigonometric C- and S-functions and specific exponential E-functions. Their properties follow from their restrictions to the fundamental domain of the affine Weyl group and-or dual Weyl group related to the dual root system.

Using standard techniques and developing new methods Jiří Hrivnák, with co-authors, derived known results and obtained various new formulas on multivariable generalizations of classical special functions. The thesis first part, on 64 pages, consists from Introduction and four chapters:

1. *Affine Weyl groups* containing relevant notions from the theory of root system of Lie algebras,
2. *Fourier-Weyl transforms*, where four types of discrete transformations are described in detail,
3. *(Anti)symmetric Trigonometric Transform*, where the symmetric exponential and cosine transformation are outlined, and
4. *Modified Multiplication of Honeycomb Transforms*, containing formulas for modified multiplication of orbit functions and where, the generalized Kac-Walton formula is summarized.

After that, follow Conclusions containing the discussion of main results together with brief comments to some open problems. The first part ends with a systematic list of References. This part is well written, rigorous, consistent and transparent.

Important second part of the thesis is represented by 9 selected publications [A1]-[A9]. Among other results, they contain 6 sections and 16 theorems and propositions with the essential contribution of the author. Their list is presented at the end of Introduction.

All papers included in the habilitation were published (in collaboration with famous experts in the field J. Patera, M. A. Walton, L. Motlochová, J. Czyzycki and I. Kashuba) in prominent journals on mathematical physics: Journal of Mathematical Physics - 4x, Journal of Physics A: Mathematical and Theoretical - 4x and SIAM Journal of Numerical Analysis - 1x. It should be pointed out that the list of J. Hrivnák publications is considerably broader with a quite remarkable response in the math-phys community.

Plenty of interesting results are collected in Conclusions. They can be characterized as follows:

- New theorems, propositions and statements;

- Known statements and formulas but their alternative, eventually, simpler proof in a new context;
- Various results that can find and found application in physics or technology.

I shall not repeat them here in detail. comments can be found in Conclusions and details in the attached papers. Instead of that I formulate two questions, perhaps best suited to general discussions:

1) My first question is rather academic: "What result, the theorem or formula, presented in the thesis the applicant considers as most interesting or important?"

2) The second question represents my personal interests: "What part of constructions or results can be extended or adopted to particular unitary representations of simple non-compact groups?

Conclusion: Jiří Hrivnák investigated an interesting and important problem related to the theory of classical compact Lie group, namely, the multivariate Fourier-Weyl transformations and their applications. The Introduction is well written, rigorous and transparent. The attached articles contain various new results and applications. They have been published in distinguished journals oriented to mathematical physics. Some of them were written with most prominent co-authors in the field (similarly as some others papers not included into the habilitation thesis). This clearly indicates that Jiří Hrivnák has been considered as an effective and competent coworker.

Therefore I evaluate the **Habilitation with distinction**, and I suggest to the committee to let Jiří Hrivnák pass the Habilitation and to become the **Docent**.

Bratislava, 27.03.2019

Peter Prešnajder