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Evaluation of the habilitation thesis

A General Framework for Mathematical Fuzzy Logic

by Petr Cintula

The topic of this thesis, Mathematical Fuzzy Logic (MFL), is an area of research that applies techniques and methods from model theory, proof theory, universal algebra, algebraic logic and other subfields of mathematical logic to a wide range of many-valued logics that are connected to Zadeh's well known concepts of fuzzy sets and approximate reasoning in various ways. The pioneering work of Hájek, Esteva, Godo, Montagna, and many others in the 1990s brought a rigorous mathematical outlook to this family of logics, that is of interest also due to their actual and potential applications in computer science. This thesis aims at providing a general framework for MFL, that allows one to connect, generalize and extend many important, more specific results. Moreover it aims at a better understanding of the underlying logical principles in the wider context of substructural logics. It does so by skillfully applying notions and techniques from abstract algebraic logic to the class of logics concerned. Many shared characteristics and characteristic general properties of fuzzy logics thus become visible, supporting a unified approach to important problems and challenges in the area. The corresponding results, that sum up previous results of Petr Cintula (in many cases in collaboration with Carles Noguera) show a high level of mathematical sophistication and are consistently presented with exemplary mathematical rigor and clarity. In this manner the guiding thesis that fuzzy logics should be viewed as logics of chains is defended convincingly.

In the following I will shortly comment on each of the seven chapters of the thesis.

Chapter 1 briefly explains MFL by alluding to philosophical motivations that entail a particular view on fuzzy set theory and many valued logics. The need for a general theory of MFL and its proposed form and content is motivated in a manner that shows that Petr Cintula is well acquainted with the wealth of relevant literature and able to develop fruitful new perspectives, that are supported by non-trivial mathematical facts.

Chapter 2 reviews central notions, like consequence operators, variants of completeness, Leibniz congruence, etc. More particular, it provides the necessary background in abstract algebraic logic, tailored to the family of Weakly Implicative Logics (WILs), that remain in the focus throughout the thesis.

Chapter 3 investigates WILs within the context of substructural logics, a broad class of logics that naturally arise from a proof theoretic perspective. Logic SL is introduced as the weakest logic of the relevant type. A careful and extensive study of syntactic, semantic and algebraic properties of WILs clarifies their place within the rather diverse and vast family of substructural logics. As a highlight of this kind of investigation, logic SL turns out to coincide with bounded non-associative full Lambek logic. (Full Lambek logic can be seen as the basic 'corner stone' of substructural logics, on top of which all other substructural logics are located.)

In **Chapter 4** disjunction, in a generalized form, takes center stage. Disjunction may not necessarily be expressed by a single connective or even by a single schematic formula in the relevant logics. Rather a notion of generalized disjunction, based on sets of formulas, is introduced, that allows one to study the (weak) Proof by Cases Property. A non-trivial (branching) 'disjunctional' hierarchy of logics is identified. Also various characterizations of the appropriate form of 'disjunctionality' are offered. This generalizes results for finitary logics to a much wider class of logics. Another result applies the developed general concept of disjunction to the problem of axiomatizing positive universal classes of reduced matrices.

Chapter 5 develops the topic of completeness with respect to linearly ordered matrices, leading to the notion of weakly implicative semilinear logics, formerly introduced by Cintula as weakly implicative fuzzy logics. Based on the demonstration of a number central properties, various connections between semilinear implications and general disjunctions, (as developed in Chapter 4) are highlighted. Finally completeness properties are studied for refined semantic structures, i.e. for specific classes of linearly order matrices.

While Chapters 1 to 5 are devoted to propositional logics, **Chapter 6** lays the foundation for first-order predicate fuzzy logics (again understood here as semilinear logics). Not just one, but two natural variants of predicate logics emerge: minimal ones, that are complete with respect to all matrices, and stronger ones that are complete with respect to linearly order matrices. It is shown that in contrast to the propositional case, these two types of predicate logics do not coincide in general. Central properties are investigated and, most importantly, adequate axiomatizations of those logics are presented.

Chapter 7 indicates the papers and books on which the various notions

and results of the previous chapters are based. It also points out connections to other publications and provides hints at further related literature.

Summing up, this thesis not just presents isolated theorems that solve particular technical problems, but rather develops a general perspective on mathematical fuzzy logic in a stringent and convincing manner. Throughout the thesis a very high level of mathematical rigor and sophistication is maintained. Moreover, Petr Cintula shows an extensive and deep knowledge of relevant results and techniques from different relevant areas of logic. Finally, I also want to emphasize the didactic value of the elegant exposition, frequently supported by carefully chosen examples of rather abstract and subtle technical concepts. As a consequence the author is able to highlight the maturity of Mathematical Fuzzy Logic with respect to the wider context of substructural logics in an admirably solid and transparent manner. I therefore evaluate this thesis to be a significant achievement that satisfies highest scientific standards. In my opinion, it also demonstrates that Petr Cintula is actually among the very best researchers in his field. Any university that can count him among its associated researchers can be proud of that fact.

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