Report on the habilitation thesis

Multivariate Fourier–Weyl transforms and their applications

by Ing. Jiří Hrivnák, Ph.D.

The thesis under review consists of an introduction, four chapters, conclusion, the reference list and the list of Applicant's publications the thesis is built upon.

The thesis is essentially based on the following nine papers by the Applicant:

- On discretization of tori of compact simple Lie groups
- Generalized discrete orbit function transforms of affine Weyl groups
- On E-discretization of tori of compact simple Lie groups
- On E-functions of semisimple Lie groups
- Two-dimensional symmetric and antisymmetric generalizations of exponential and cosine functions
- Discrete transforms and orthogonal polynomials of (anti)symmetric multivariate cosine functions
- Discretized Weyl-orbit functions: modified multiplication and Galois symmetry
- Weight-lattice discretization of Weyl-orbit functions
- Discrete cosine and sine transforms generalized to honeycomb lattice

While many of the Applicant's publications are co-authored, the diversity of coauthors in academic standing, geography and stature provides quite a credible evidence for the research independence of the Applicant. In particular, for the nine papers listed above

- the first, third, and fifth papers are joint with J. Patera (Montréal, Canada)
- the second paper is joint with T. Czyżycki (Białystok, Poland)
- the fourth paper is joint with I. Kashuba (São Paulo, Brazil) and J. Patera (Montréal, Canada)
- the sixth and ninth papers are joint with L. Motlochová (Prague, Czech Republic)
- the seventh and eighth papers are joint with M. Walton (Lethbridge, Canada)

I would be glad to learn what motivated the Applicant's choice to build the thesis around this particular set of papers given that he has quite a number of other publications.

With all of the above nine papers making notable contributions to their subject area, I would particularly like to single out the first one, *On discretization of tori of compact simple Lie groups* which presents three types of certain numerical data for simple Lie groups of any type and rank. The said data are then employed for Fourier-like expansions of multidimensional digital data into finite series of C- or S-functions on the fundamental domains of the underlying Lie groups.

I also quite like the seventh paper, *Discretized Weyl-orbit functions: modified multiplication and Galois symmetry*, uncovering an interesting similarity between affine modular data associated with Wess–Zumino–Novikov–Witten (WZNW) conformal field theories and discretized Weyl orbit functions. This similarity allowed the authors to obtain certain important results concerning the orbit functions in question.

As far as the specific results of the above nine publications are concerned, I would also like to highlight in particular Theorem 3.3 in the second paper and Theorem 4.1 in the eighth paper, as they establish rather useful facts on the geometry of the Weyl groups associated to certain Lie algebras.

Having said that, I would also like to point out that in the papers constituting the thesis one finds not just new theorems but also a number of useful new definitions introducing new kinds of transforms related to the orbit functions.

The author of the thesis is a productive researcher, who has overall authored more than 30 publications to date, predominantly in international journals of very good repute. For example, the sixth of the nine papers listed above is published in the high-quality journal of the Society for Industrial and Applied Mathematics (USA), *SIAM Journal of Numerical Analysis*. Further evidence of the man's research activity is the fact that, to the best of my knowledge, he now supervises two Ph.D. students, Adam Brus and Michal Juránek.

To complete the general description of the Applicant's research profile let me point out that upon the defense of his doctoral thesis he secured a postdoctoral fellowship at the renown Centre de Recherches Mathématiques of the Université de Montréal, Canada, under the auspices of a highly prominent scientist, prof. Jirí Patera.

Now, to get back to the thesis itself, the main task of the introduction and conclusion is, quite naturally, to put the Applicant's research presented in the thesis into a broader context, including the applications.

Next, the first chapter is also of introductory nature: it sets up the notation and reviews a number of concepts from the theory of Lie algebras and associated Weyl groups and affine Weyl groups required for the rest of the thesis.

The second chapter, touching upon the first four of the nine papers listed at the beginning of the present report, starts with defining one of the key protagonists of the thesis, the orbit functions, and proceeds to define the discrete transforms related to the latter.

The third chapter deals with symmetric and antisymmetric exponential and trigonometric transforms. The Applicant then proceeds to present the applications of the transforms in question to the interpolation problems in two and three independent variables establishing inter alia the uniqueness of interpolating functions under certain natural assumptions. The chapter closes with a section discussing the cosine cubature formulas.

The fourth chapter explores the connection between the Fourier–Weyl transforms and conformal field theory, including the link among the Kac–Peterson matrices from the latter and the weight lattice Fourier–Weyl transforms, as well as the applications of orbit functions to solid state physics, including the so-called transversal vibration models. Also, the multiplication formulas for orbit functions and the related Galois symmetries are touched upon.

The thesis is not free of drawbacks. For one, introduction and conclusion could, in my opinion, be a bit less technical and do a somewhat better job of placing the results of the Applicant in the big picture of the subject. It would also be nice to see a somewhat more detailed discussion of connections of the Weyl groups to the representation theory in Chapter 1. There also is a number of misprints, e.g. *coefficents* instead of *coefficients* at p. 10, or the publication year 1990 instead of 1997 in [26] at p. 56, the missing volume, issue and page numbers for the reference [87] at p. 60, and *Chebyshef* instead of *Chebyshev* in [95] at p. 61.

To conclude, the above critical remarks notwithstanding, in my opinion the thesis meets the requirements of Act No. 111/1998 Coll., on Higher Education Institutions, as amended, and clearly indicates the Applicant's aptitude for research, so I am pleased to recommend the thesis to be accepted and the habilitation process to go on.

Opava

doc. RNDr. Artur Sergyeyev, Ph.D.